David Burghes

Primary Problems

A First Curriculum for Mathematics

Curriculum Series
Edited by
Sheila Lawlor

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Foreword
Sheila Lawlor

The educational standards reached in this country remain low by comparison with those in many similar economies in Europe and others globally. Our pupils have fallen behind Germany's and have also been overtaken by their peers in some new European economies like Poland. The last round of the OECD's tests (PISA) for 15 year olds showed that the UK's ranking had fallen for literacy, science and mathematics: reading was down to 25th from 17th in 2006, science down to 16th from 14th, and numeracy down to 28th place, from the already low 24th three years earlier.[1]

To the results of the PISA tests could be added other evidence which suggests that the academic and intellectual bar for pupils and their teachers has been set far lower in this country than in similar western industrial economies We expect less from both pupils and teachers when it comes to academic subject standards than do many other countries.[2]

Since 2010, the Coalition has accelerated the pace of some reforms introduced by its predecessors and proposed additional reform. A fresh emphasis has been placed on liberalising the structure of schooling; a new school curriculum will be introduced; and the examination system will be reviewed. The aim is to ensure that our system is pitched at the level of academically high performing countries, so that the educational opportunities which can be taken for granted in competitor countries are open to all pupils here.

In line with these reforms, the DfE's draft mathematics curriculum has now been published for consultation, along with those for science and English. The government intends its proposals 'to set out the very highest expectations for all pupils and to be on a par with the highest-performing education jurisdictions in the world'.[3]

Politeia's curriculum series responds to these developments. It aims to inform the important discussion of how and how far the state should determine the content of what is taught with individual proposals from academic subject specialists. It will also

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[1] *Programme for International Student Assessment*, (2009), http://stats.oecd.org/PISA2009Profiles/# Pisa. *The Guardian*, 10 Dec 2010.

[2] For mathematics, see below pp. 10-11, 13-14. The comparative academic standards expected of teachers and school leavers respectively were considered in the decade up to 2010 by Politeia's *Comparing Standards* Series: *Teachers Matter: Recruitment, Employment and Retention at Home and Abroad*, D. Burghes, J. Howson, J Marenbon, et al. and *Academic and Vocational, 16-19*, D. Burghes, A. Smithers, R. Tombs, et al.

[3] Department for Education, *Draft National Curriculum documents for primary English, mathematics and science*, www.education.gov.uk/schools/teachingandlearning/curriculum/a00210036/sosletter.

reflect the themes in our education series which emphasise respect both for high academic standards and teachers' professional freedom.

This mathematics curriculum, drawn up by Professor David Burghes, has paid particular attention to the lessons of Finland, Singapore and Japan, three ‘mathematically high performing countries’. In particular, he emphasises the importance of the primary curriculum as the starting point for change. He advocates learning the foundations, including number bonds and multiplication tables, early on: they are mastered by pupils in his three high-performing countries by ages 8-9.

Professor Burghes welcomes the DfE’s proposals for mastering key number facts, and for greater freedom for teachers, but proposes a number of refinements for the DfE’s scheme: some parts of the curriculum should be introduced earlier than proposed, and others later.

He concludes by setting out a clear, short curriculum for each year of the primary phase, illustrated with exemplar questions, in a format which will be helpful to teachers and welcomed for its brevity and clarity.

Sheila Lawlor
Director, Politeia

I
Introduction: School Mathematics Today

Mathematics remains a central subject in all sectors of education and will be part of the new national curriculum, due to be introduced in 2014, with draft proposals now out for consultation. However, teaching mathematics remains controversial. Sir Simon Jenkins, the columnist, considered his own mathematics study 'a waste of time' and regrets the 'millions of pounds … [spent on] teaching maths to reluctant pupils …' Elizabeth Truss MP took the opposite view as a backbencher.[4] This study takes the view that mathematics is an important and relevant subject, necessary to lead a full adult life. It proposes an effective curriculum for primary mathematics. But it does not recommend more mathematics in UK schools until a higher standard of teaching can be provided for good, effective and inspirational mathematics teaching throughout the entire education system.

Section 2 reviews the current problems in mathematics education in the UK with special focus on England. In recent years there has been renewed interest in mathematics teaching in the mathematically high-performing countries (MHPCs) around the Pacific Rim. Many of these countries have been characterized by high attainment in examinations (and in international comparisons) and are now working to develop their students as mathematical thinkers, able not only to perform well in mathematical tests, but also to use and apply their mathematical knowledge in practical and new contexts. Details of these developments, including the pupil age when key topics are introduced in the MHPCs, are given in Section 3.

A comparison is made in Section 4 with the Department for Education's draft National Curriculum and the differences we see as significant between these two curricula.[5] In Section 5, recommendations are made for the English Primary Mathematics National Curriculum, reflecting international practice in the MHPCs.

Provided a period of stability were to follow, we think that a revised national curriculum for primary mathematics, designed to ensure that pupils would enjoy and appreciate the subject, improve their mathematical knowledge and skills and their ability to use and apply these skills, would provide long-term benefit for the country in a highly competitive world.

[4] Simon Jenkins, *The Guardian* 6 June 2008 www.guardian.co.uk/commentisfree/2008/jun/06/maths.alevels See also comments made by Elizabeth Truss MP, June 2012, www.bbc.co.uk/news/uk-england-18543544

[5] Department for Education, *Draft national curriculum documents for primary English, mathematics and science*, 2012, www.education.gov.uk/schools/teachingandlearning/curriculum/a00210036/sosletter.

II
The Problems

A number of factors appear to hold back the teaching and learning of mathematics in England. Despite significant investment in primary mathematics since the National Numeracy Strategy (NNS)[6] was introduced more than a decade ago, there are doubts about progress. The recent Cambridge Review questions whether significant gains have been made.[7] Although some evidence suggests that mental skills have improved and many pupils do well at Key Stage 2 (age 11-14), nonetheless many have not reached an adequate standard by the time they leave primary school. There is great variation in attainment at the end of Key Stage 2, first identified 30 years ago![8] Little seems to have changed despite the best efforts of teachers, advisors and government initiatives.

The quality of the mathematical knowledge of teachers remains an important influence. The majority of entrants to the profession have only a GCSE Grade C as their highest mathematical qualification. The emphasis throughout primary is on numeracy and accessible topics in, for example, shape and space, rather than the provision of a mathematical foundation (including algebra) on which to build in the secondary sector.

In the secondary sector the consequences must be addressed. The first is how to deal with an entry cohort of pupils with such a wide range of attainment. Often, teachers start again, which may work for some but can be boring and tedious for those with high mathematical achievement at primary. Most schools 'set' their classes according to ability early in Year 7 and whilst this can deal with the wide spread of attainment, it can also increase the gap between the 'high fliers' and the lower attainers, who often lose interest and motivation by appearing to fail at the outset of secondary education. As happens in other subjects, many candidates find the current provision boring, partly because of the continual repetition of topics throughout the Key Stages.

There is also a problem with the retention and recruitment of teachers. The turnover is high given that approximately 2,500 new secondary mathematics teachers are trained each year yet there are only about 12,000 secondary mathematics teachers in the

[6] Department for Education and Employment, *The National Numeracy Strategy: framework for teaching mathematics from reception to Year 6*, 1999.

[7] R. Alexander ed., *Children, Their World, Their Education: Final Report and Recommendations of the Cambridge Primary Review*, 2010.

[8] *Mathematics Counts*, Report of the Committee of Inquiry into the Teaching of Mathematics in Schools under the Chairmanship of Dr WH Cockcroft, 1982, www.educationengland.org.uk/documents/cockcroft/.

maintained sector. The mathematical standard at entry to the profession is very varied.[9] Furthermore, the majority of secondary mathematics teachers do not remain in post even for five years. Yet research indicates that it takes about 5 years for a newly-qualified teacher to become an expert teacher. There are, of course, many explanations for this poor retention rate, including:

- migration to the independent sector.
- promotion out of mathematics teaching, remaining in education.
- career opportunities for secondary mathematics teachers outside school (including in high-level technology companies).
- dissatisfaction with employment conditions, particularly the amount of paperwork required (rather than time spent teaching).
- poor behaviour of classes, particularly the middle and lower sets.
- the lack of trust in schools; where OFSTED, or rather the fear of OFSTED, dominates and teachers are aware that they are judged on results.

For these and other reasons, we lack and/or lose too many talented and creative teachers who are mathematically well-qualified and subsequently schools are forced to make unwise appointments solely to ensure that classes have teachers.

Different ideas have been canvassed for remedying the poor supply of mathematically competent school leavers and teachers. The possibility of double subject mathematics at GCSE has been mooted – with a more academic course for those intending to continue mathematics at a higher level and a more practical course for those intending to continue more vocationally-based courses. A recent suggestion has been that O-level Mathematics will be re-introduced as part of an English Baccalaureate; and though this has caused a stir, it might in practice not be very far away from the double subject initiative.

For mathematics in the secondary years, one problem is that the current syllabus for GCSE Mathematics (and similarly for Key Stage 3 Mathematics) does not reflect the dynamic and practical aspects of the subject. For example, new technology depends on mathematics, particularly the binary mathematics that underpins much of the recent developments in applying mathematics, so the subject could involve subjects that would be relevant and inspiring to students. Topics such as critical path analysis, linear programming and error-predicting codes could have a place in a contemporary, applicable curriculum for practical mathematics during Key Stage 4 (ages 14 to 16).

[9] CfBT Education Trust , *International comparative study in mathematics teacher training*, 2011, www.cfbt.com/evidenceforeducation/our_research/evidence_for_government/international_policy_reforms/current_research_projects/maths_comparative_study.aspx.

The 16-18 stage is more successful despite the problems already mentioned. Most sixth form teaching is with classes of well-motivated students who want to do well in their AS/A level modules. This brings a good working ethos. Helped by publicity about the value of A-level Mathematics and Further Mathematics, the numbers taking both A-level Mathematics and A-level Further Mathematics have been rising. This is particularly the case for Further Mathematics. Numbers are not, however, back to the percentage share levels seen several decades ago.

The upshot is that despite the increase there are not enough graduates with either mathematics or some component of mathematics in their degrees and even during a time of recession, there continues to be a serious shortage of well-qualified and talented mathematics teachers, particularly in the secondary sector.

The causes and effects of these shortages of mathematically educated people throughout the system are well illustrated in the loop used in a 2011 report on mathematics education.[10] We have too few teachers at primary school with a real understanding of mathematics, leading to children not being fully extended; the pupils continue into the secondary stage, where there is a shortage of adequately trained mathematicians so that many of them are either bored or not coping. This results in not enough students in the sixth form taking mathematics and low numbers of students undertaking mathematics or mathematics-related degrees at universities. The cycle continues with not enough mathematically well qualified young people entering the teaching profession.

A route must be found to break this sequence. It may seem easier (or provide quicker results) to concentrate on secondary school mathematics, but for long-term sustainable enhancement, the aim must be to change primary mathematics. This is the only way to improve standards of attainment and the attitudes to mathematics. Therefore, the focus of this study is to propose the principles and approach to guide the new primary maths curriculum and to provide the specialist evidence from comparator countries which the government intends should in general be used to guide the standards to which we should aspire in this country.

[10] C. Vorderman et al., *A world-class mathematics education for all our young people*, 2011, p.11, www.tsm-resources.com/pdf/VordermanMathsReport.pdf.

III
The Mathematically High-Performing Countries (MHPCs)
Curriculum Lessons for England and Wales

This section considers a group of mathematically high-performing countries – Finland, Japan and Singapore.

It should be stressed that these three countries employ a 'mastery' type of curriculum in which they expect that almost all pupils will master the yearly topics and be ready to move on. The concepts will, of course, be reinforced and revised in subsequent years but it is assumed that most of the pupils will have 'fixed' the concepts when they are taught. Each of the countries is considered separately and a series of significant characteristics highlighted.

Finland

Finland is a culturally cohesive society of just over 5 million, mainly concentrated in Helsinki and the south west of the country and with children attending their local primary schools where mathematics and other classes are not streamed or set at primary or indeed secondary level. Mathematics lessons begin at age 7 and by age 9, most pupils will have mastered topics including number bonds, addition and subtraction, up to 20 (for example, for 10, 0+10=10, 1+9=10, 2+8=10, … 10+0=10), multiplication tables up to 10x10 and the use of mathematical symbols, including inequality signs. Teachers have a high mathematical standard of knowledge and skills.[11]

Finland's education system has been overhauled over the past 25 years, with 'equity' as the underlying theme for success. In fact, recent success in international comparisons, not only in mathematics but also in science and native language, are a bi-product of the equity agenda, rather than its aim. Teachers and educators in Finland (see below note 10) seem shy about their success, explaining that the aim was equity for all children rather than high scores on international tests. Rather than detail the changes over the past 2-3 decades, the current situation and practice in mathematics is considered.

The current practice is characterised by the following points:

- The population spread is 20 per cent in Helsinki, 65 per cent in the other towns and 15 per cent in rural communities. Pupils attend their local school (with

[11] Ole Bjorkqvist, *Mathematics education in Finland – what makes it work?,* 2005. http://math.unipa.it/~GRIM/21_project/21_malasya_Bjorkqvist45-48_05.pdf.

good schools provided nationally and no competition between schools). Parents are therefore confident their children will obtain a good education and be given the opportunity to succeed. There are almost no private schools (the exception being International Schools in Helsinki) and no call for them, given the high standards in the state sector.

- Extra funding is given to schools in deprived areas to ensure that the 'equity' provision is fulfilled.
- Classes are highly integrated with only 2 per cent of pupils attending special educational institutions.
- There are no national exams until age 18, although teachers continually monitor the progress of the students in their classes.
- Schools have the feel of a 'second home' for children; they remove their shoes when they enter, receive a free hot meal every day and are given both freedom and trust.
- Teachers, who are well educated for a highly competitive entry to the profession, are regarded as professionals, given freedom to innovate in any way they wish (they also receive a free hot meal every day); the ratio of applicants for primary training is approximately 10 : 1.
- There is no schools inspection regime but a supportive education service is provided.
- Teachers' higher mathematics education is developed at school level, with all students taking some form of mathematics throughout the equivalent of our sixth form study. Teacher training takes place in universities using University Practice Schools for both observations and teaching practice (these are state schools which function in their prescribed catchment area but with a reduced student to teacher ratio to reflect the demands of working with numerous trainee teachers; classrooms are large enough to accommodate groups of trainee teachers to observe). It is clear that the conditions of service for teachers are somewhat different from those in England – and teaching as a career is very popular, although it is not particularly well paid.

The Finnish National Curriculum for Primary Mathematics presents a good summary of the constraints put on teachers.[12] It gives an outline of what should be covered in this phase and requires a foundation in mathematics which, by age 11, most students have mastered. It allows freedom for teachers to use their own preferred methodology. In many, but not all, cases, the methodology is student-focused with activities forming a substantial part of lessons in which the students, working in pairs or groups, develop or apply mathematical thinking to problems set by their teacher. Students can take

12 Details of the Finnish National Curriculum for Mathematics are given on the website www.cimt.plymouth.ac.uk/politeia/mathematics.

responsibility for their learning, with extension work for the most able and extra support for the less able. Note that:

- in Grades 1-2 (ages 7 - 8), the development of mathematical thinking is given prominence along with basic arithmetic skills including multiplication, division and multiplication tables
- in Grades 1-2, dealing with data is included
- in Grades 3-5 (ages 9 - 11), there is a significant component in algebra, including algebraic expressions and solutions of equations and inequalities
- also in Grades 3-5, measures of central tendency, that is, mean, mode and median, are included as well as classical and statistical probability.

Mathematics classrooms are cheerful, orderly places with little disruption or indiscipline. Education is valued by students, parents and staff; students are trusted to work and teachers trusted by parents to provide their children with a good education. Although very different from the experience in England, Finland's education programme, built on a homogenous society and equality and trust, provides a high standard of education that may have lessons for our own system, particularly in terms of trust.

Finnish teachers are well qualified, dedicated to their profession, enjoy teaching and relate well to their students.[13]

Japan

Japan is a culturally cohesive society of 128 million people, mainly concentrated in towns and cities, with children attending their local primary schools where mathematics and other classes are not streamed or set. Mathematics lessons begin at age 6 and by age 9, most pupils will have mastered topics such as number bonds, multiplication tables, algebraic symbols and addition and subtraction of fractions. Mathematics teachers, both primary and secondary, have very high standards of knowledge and skills,[14] illustrated in the table below.

Japan's educational system has similar characteristics to Finland's, with pupils attending their local school and with University Practice Schools an integral part of the state sector as well as the teacher training system. There are two important additional strategies:

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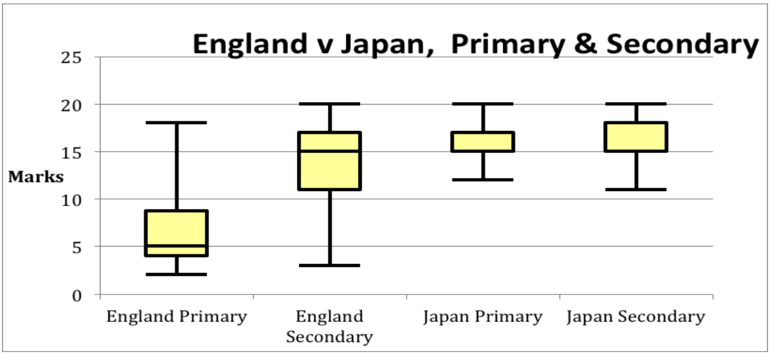
[13] For a more detailed account of the educational route that Finland has taken over the past three decades, see OFSTED, *Finnish pupils' success in mathematics,* 2010.

[14] CfBT Education Trust , *International comparative study in mathematics teacher training*, 2011, www.cfbt.com/evidenceforeducation/our_research/evidence_for_government/international_policy_reforms/current_research_projects/maths_comparative_study.aspx.

- Teachers are allocated to a region, not a school, and can expect to move schools every 3-5 years (this also applies to Heads of schools) and therefore face new challenges. The authorities can move expert teachers into any school which needs extra help.
- The main element of CPD (continuing professional development), particularly in primary schools, is lesson study where groups of 3 or 4 teachers meet together to plan a 'research' lesson designed to meet a number of specific objectives resulting from the school's overarching aims for mathematics (an example is given in Appendix 1). One member of the group gives the lessons, observed by the others and then the group reviews and evaluates the lesson, making changes to the lesson plan where appropriate. This process often involves a staff member from the local University Education Department.

Lesson study is thought to be one of the reasons for Japan's enhanced scores in recent international comparisons. CIMT's international teacher training research project[15] showed that the mathematical skills and knowledge of Japan's teachers are very high. The box and whisker plots shown below illustrate the mathematical attainment of primary and secondary future teachers in England and Japan in a set of basic mathematics questions. See Appendix 6 for the set of common questions taken by samples of trainee teachers as part of this research project. In these questions, the Japanese teachers perform significantly better than the English teachers.



Note that:

- Japan out-performed England overall and did not have the 'tail' of low performance exhibited by both our primary and secondary trainee teachers.
- the performances of the Japanese primary and secondary samples are almost identical, illustrating the high mathematical qualification of *all* these teachers.

15 Ibid.

As well as achieving highly in mathematical skills and knowledge, Japan, along with other Pacific Rim countries, has embarked on an ambitious project to develop their students' skills in applying their mathematical skills in contexts and to new situations whilst maintaining high standards of mathematical knowledge. To this end they have revised the Grades 1-6 curriculum and published a detailed Teaching Guide.[16] The stated objectives for mathematical instruction are:

> Through mathematical activities, to help pupils acquire basic and fundamental knowledge and skills regarding numbers, quantities and geometrical figures, to foster their ability to think and express with good perspectives and logically on matters of everyday life, to help pupils find pleasure in mathematical activities and appreciate the value of mathematical approaches, and to foster an attitude to willingly [sic] make use of mathematics in their daily lives as well as in their learning.
>
> [Mathematical activities] can include various activities. Hands-on activities, experimental activities, physical activities, and activities that use concrete objects are often considered to be typical examples of mathematical activities. But there are others.
>
> Thinking about mathematical problems, building on mathematical knowledge and applying that knowledge, representing and explaining what students think – these do not deal with concrete objects but are included in mathematical activities.

The Teaching Guide also states that 'when students just listen to teachers' explanations or complete practice problems', these are not regarded as mathematical activities; note though that Japan in this way is trying to counter a traditional culture of deference in learning, which may not apply to Western societies or indeed our own. These reforms have been implemented. They appear to have succeeded in enabling students to 'find pleasure in mathematical activities' whilst continuing to develop and achieve in number work. A recent example observed earlier this year in a Japanese classroom involved 29 primary school students aged 10 (yr 4) being asked to write a whole number between 1-99 and then to classify the numbers on the board. The details are in the illustrative box below. This was one of three parallel classes of mixed ability students with no streaming or setting.

The main focus was reinforcement of the odd/even classification for numbers; this was expanded upon and developed towards the end of the lesson, with more classifying examples being shown and discussed.

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16 Masami Isoda ed., *Elementary School Teaching Guide for the Japanese Course of Study: Mathematics (Grades 1-6),* http://science.kennesaw.edu/~twatanab/Teaching%20Guide%20Elementary.pdf.

The lesson opened with each of the 29 students being given a card and asked to write on it a whole number between 1 and 99. Each student then placed their (magnetic) card on the board at the front of the class. (Note that, as the students had chosen and written the numbers themselves, the teacher immediately had their attention and interest. Also, he had no way of knowing the numbers that would be chosen and, at this stage, it was not clear to the students (or the observers) what the objective of the lesson was.)

The students were directed in the next phase to work in pairs to classify the numbers on the board in any way they could think of. The numbers on the board are shown below.(Note that the fact that the cards were magnetic meant that they could easily be moved on the board, and this is indeed what happened when each pair of students illustrated their chosen classification.)

The teacher also considered the 'doubles' and showed that the number of students in the class could be calculated from $7 \times 4 + 1 = 29$ He then asked each pair of students to classify the set of numbers in any way they chose. One of each pair then presented their classification on the board.

Examples included

- numbers that appeared once (or twice)
- multiples (and non-multiples) of 11
- prime (and non-prime) numbers
- one-digit (and two-digit) numbers
- odd (and even) numbers.

The stated Japanese Course of Study (as with Finland), from which schools develop their own curricula, is commendably brief, with teachers again having freedom to innovate and extend the curriculum.[17] It should be noted that:

- throughout, but starting in Grade 1, 'mathematical relations' are emphasized by using algebraic expressions
- similarly, and again starting in Grade 1, mathematical activities are stressed and, in Grade 1, used to reinforce counting, calculation, comparison, shapes and algebraic expressions
- in Grade 2, multiplication tables up to 9x9 are covered
- fractions, including simple addition and subtraction, are introduced in Grade 3; multiplication and division of fractions in Grade 5
- decimals are introduced in Grade 3 and developed throughout Grades 4 and 5.

[17] An outline is given on the website www.plymouth.ac.uk/politeia/mathematics.

We finish with another example, used with Grade 3/4 pupils, that illustrates both the mathematical thinking that is encouraged and how this is put into a meaningful context.

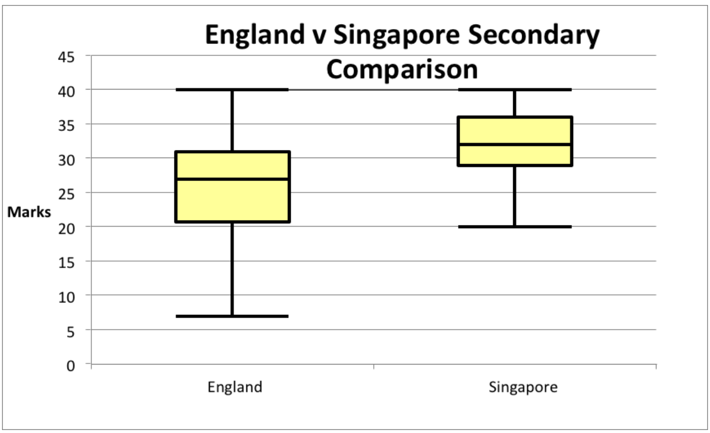
> A cinema has 100 seats. Show how it is possible to sell exactly 100 tickets and take exactly £100 if the prices are:
> £10 for adults 50p for pensioners 10p for children
> Is there only one solution?

Singapore

Singapore is a culturally cohesive, diverse society with a population of just over 5 million, mainly concentrated in Singapore City and the surrounding towns with children attending local primary schools at age 6 where mathematics and other classes are not streamed or set for the first few years. There is a choice of schools and even more choice in the secondary sector. Mathematics lessons begin at age 6 and by age 9, students will have mastered, as in Japan, topics such as number bonds, multiplication tables, algebraic symbols and fractions.

Mathematics teachers have high standards of knowledge and skills as the diagram below shows for the secondary sector, where Singapore's maths teachers did significantly better when tested on maths questions than their English counterparts. As can readily be seen, the box and whisker plots for Singapore and England trainee teachers show that:

- the median score of Singapore is significantly more than England.
- the variation for Singapore is also significantly less than England.



Trainee teachers in Singapore outperform their counterparts in England. There is not the long 'tail' of teachers who do not even achieve an adequate standard in mathematics.

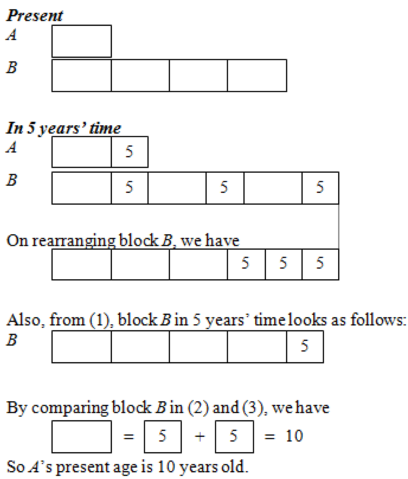
Singapore follows a similar pattern to Japan, although with some differences. The stated aims for mathematical education in schools and the principles on which Singapore's primary maths teaching is based are: Skills Concepts, Processes, Attitudes and Metacognition.

'Mathematical Problem Solving' is at the heart of the framework used for teaching mathematics.[18] Government and educators are keen to build on Singapore's success in external examinations (more that 80 per cent of a yearly cohort pass O-Level examinations, and the modal grade for A-level mathematics is Grade A). This framework reflects the underlying principles of an effective mathematics programme that is applicable to all levels, from the primary to A-levels. It sets the direction for the teaching, learning and assessment of mathematics. It reflects that mathematical problem solving is central to mathematics learning. It involves the acquisition and application of mathematics concepts and skills in a wide range of situations, including non-routine, open-ended and real-world problems.

To help primary teachers in particular to begin to use algebraic methods, they employ what they call the 'Box Method' for solving problems in context. Here is an illustrative example of how this works:

A is one quarter as old as B. In 5 years' time, A will be one-third as old as B. How old is A now?

Solution: We will approach this problem, using boxes, as follows:



This is an alternative to the algebraic method of solving the problem – see Fig. 1 in Appendix 3.

[18] Ministry of Education, Singapore, *Primary Mathematics Syllabus*, 2006, www.moe.gov.sg/education/syllabuses/sciences/files/maths-primary-2007.pdf.

The Singapore National Curriculum for Primary Mathematics is a more detailed document so we have included on the website just the important first 3 years (Primary 1, Primary 2 and Primary 3).[19] Some interesting points to note include:

- use of the inequality symbols (> and <) starts in Primary 1.
- number bonds up to 9 x 9 in Primary 1 are 'committed to memory'.
- mental calculations are stressed throughout.
- Data Analysis is introduced in Primary 1.
- multiplication tables, 2, 3, 4, 5 and 10, are committed to memory in Primary 2.
- fractions, including simple addition and subtraction, are introduced in Primary 2.
- time and money, and also length, mass and volume, are used in Primary 2.
- multiplication tables up to 9 x 9 are all committed to memory in Primary 3.
- area and perimeter are introduced in Primary 3.

Finland, Japan and Singapore have one factor in common: the continuum between primary and secondary mathematics. The foundations for mathematics are introduced throughout the primary phase with, for example, early algebraic foundations beginning to be introduced from the very first year of primary education.

Not only is the correct use of the equals (=) sign stressed, but $\neq$, $<$, $\leq$, $>$, $\geq$ are all introduced and used correctly in the first years of teaching, together with precise definitions.

An example from Year 1 is the problem of solving, 7 + ? = 10,
where ? is the unknown. If pupils know their number bonds, they will have no difficulty giving the answer (3) but teachers with mathematical understanding would use problems like this to illustrate the fundamental concepts of algebra. For the pupil response of "3", they would ask, "Why?", and expect to be given an answer such as,

"Because 7 + 3 = 10" or "10 – 7 = 3" or even "10 – 3 = 7"

By extending responses in this way they are stressing the use and importance of the equals sign and embedding the basics of solving linear equations.

[19] Details of the full document that includes an emphasis on the framework for teaching and learning mathematics are given at www.plymouth.ac.uk/politeia/mathematics.

Conclusion and Summary

Finland, Japan and Singapore employ a mastery curriculum in which they expect that almost all their pupils will master the yearly topics and be ready to move on.

It is of interest to note the age at which formal primary education starts in each of these MHPCs, and when some of the key mathematical topics are introduced in comparison with the proposals in the draft National Curriculum. The table below gives a summary of these points.

	FINLAND*	**JAPAN**	**SINGAPORE**	**ENGLAND AND WALES**
Starting age	7 (Grade 1)	6 (Grade 1)	6 (Primary 1)	5 (Year 1)
Number bonds (up to 20)	Grade 1/2 (Age 7-9)	Grade 1 (Age 6-7)	Primary 1 (Age 6-7)	Year 1 (age 5-6)
All multiplication tables (up to 10)	Grade 1/2 (Age 7-9)	Grade 2 (Age 7-8)	Primary 3 (Age 8-9)	Year 3 (age 7-8)
Inequality symbols	Grade 1/2 (Age 7-9)	Grade 3 (Age 8-9)	Primary 1 (Age 6-7)	Year 2 (age 6-7)
Fractions: addition and subtraction	Grade 3/5 (Age 9-11)	Grade 3 (Age 8-9)	Primary 2 (Age 7-8)	Year 3 (age 7-8)
Fractions: multiplication and division	Grade 3/5 (Age 9-11)	Grade 5 (Age 10-11)	Primary 5 (Age 10-11)	Year 6 (age 10-11)
Decimals: addition and subtraction	Grade 3/5 (Age 9-11)	Grade 3 (Age 8-9)	Primary 4 (Age 9-10	Year 5 (age 9-10)
Decimals: multiplication and division	Grade 6/9 (Age 11-14)	Grade 4 (Age 9-10)	Primary 4 (Age 9-10)	Year 6 (age 10-11)
Percentages	Grade 3/5 (Age 9-11)	Grade 5 (Age 9-10)	Primary 5 (Age 10-11)	Year 5 (age 9-10)

***** Curriculum only specified for Grades 1/2, 3/5 and 6/9 rather than year by year

All these MHPCs have implemented revised national curricula in which knowledge of the fundamentals is still stressed but also in which problem-solving activities play a central part in the mathematical development of their pupils. This emphasis on problem-solving is complementary to that of ensuring the fundamentals are mastered, indeed, it provides a motivating way of ensuring that pupils make progress and the mathematical thinking encouraged is a way of ensuring that the fundamental topics are both mastered and fixed!

Teachers orchestrate motivational activities in the classroom. Students develop inquiring minds and a thirst for knowledge: they take responsibility for learning and how to learn! This methodology contrasts with the transmission model of teaching mathematics experienced in the UK, where teachers have the knowledge which they give to their students through examples, and pupils spend much of the remainder of the lesson in practice mode.

The methodology in these MHPCs is for the students to develop for themselves the next concept or technique, through problems set by the teacher. There is still plenty of practice and reinforcement but, as the Japanese mathematician, Professor Masataka Koyama explains, the teacher's aim is to 'fix' topics through this problem-solving methodology so that they do not have to repeat techniques and concepts each year (as we tend to do in England).[20]

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[20] CfBT Education Trust , *International comparative study in mathematics teacher training*, 2011.

IV
The DfE's Draft Curriculum
On Track?

The proposed new national curriculum was announced in 2010 and the draft proposals for mathematics were published for consultation earlier this year.[21]

Our proposals have in common with the draft an emphasis on pupils mastering key number facts, such as number bonds up to 20 **(**addition and subtraction**)**, on learning multiplication tables and on using traditional vertical calculation for numerical calculations. A welcome feature is the freedom for teachers to introduce parts of the curriculum earlier than stipulated and also to allow the inclusion of extension topics outside the stated curriculum. Finally, our evidence also supports the idea that the curriculum is given for each of the 2-yearly Key Stages rather than yearly, in order to give freedom to teachers (educators and curriculum developers) to decide what best suits their pupils and their own interests and expertise.

However, we would include some topics earlier, that is inequality signs ($<$, $\leq$, $>$, $\geq$) introduced in Year 1; multiplication tables up to 10 x 10 by the end of Year 2; Tables and Venn diagrams for use in classifying numbers/objects from Year 2 onwards; probability introduced in Year 2 and developed throughout the following years; algebraic notation and expressions used in Year 1 and throughout the primary phase, with solving equations and inequalities from Year 2 onwards.

There are just two strands in this curriculum, namely *Number* and *Geometry and Measure.* However, the absence of strands in Algebra, Probability and Handling Data is not in line with MHPCs, where these topics, particularly Algebra, are seen as important aspects of a foundation in Mathematics. Also missing is any emphasis on problem solving activities, particularly important for motivation and for using and applying mathematics and for developing the logic and rigour that underpin mathematics.

A detailed Year-by-Year comparison of the draft National Curriculum is given on the website www.cimt.plymouth.ac.uk/politeia/mathematics.

These points are discussed in more detail below.

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[21] Department for Education, *Draft national curriculum documents for primary English, mathematics and science*, 2012, www.education.gov.uk/schools/teachingandlearning/curriculum/a00210036/sosletter.

We would recommend the following are introduced <u>earlier</u> than stated in the draft National Curriculum

Inequality Signs

We recommend the inequality signs (<, >) are introduced in Year 1 (age 5-6) (rather than Year 2 (age 6-7) together with the ≤ and ≥ signs. This would reinforce the correct use of the = sign, as well as ≠, and also prepare key work towards algebra; for example,

? < 3 gives ? = 0, 1 or 2 whilst,

? ≤ 4 gives ? = 0, 1, 2, 3 or 4

Multiplication Tables

We recommend bringing in multiplication tables at an earlier stage than year 3 (age 7-8), so that by the end of Year 2, all multiplication tables up to 10 x 10 will have been introduced and reinforced.

This would enable the logic of the multiplication square to be stressed throughout, moving from

0	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6			15					30
4	4	8			20					40
5	5	10	15	20	25	30	35	40	45	50
6	6	12			30					60
7	7	14			35					70
8	8	16			40					80
9	9	18			45					90
10	10	20	30	40	50	60	70	80	90	100

by bringing in the 3, 4, 6, 7, 8 and 9 in a stage-by-stage process until completion.

Probability
Leaving the introduction of Probability until Year 6 (age 10+) seems far too late and is out of line with the MHPCs. This is a vitally important topic and one that young children can understand. It is also excellent for practice in the use of fractions and decimals and it underpins many modern-day applications of mathematics in business and commerce.

We would want to bring in the concept in Year 3 (age 7-8) rather than Year 6, and then develop the topic in both a practical and a theoretical way, encouraging a motivating, creative and practical side of mathematics. You can see this development, with some illustrative questions in our proposed curriculum in Section 4 above.

Algebra
This is a vitally important mathematical topic but is only included in Y6 in the draft curriculum; and so is out of line with the MHPCs where introductory aspects of Algebra are developed from the first year of primary education!

This takes the form of an introduction to the notation, concepts, logic and rigour that underpin algebraic developments – and indeed, all mathematics. We suggest that this approach is the natural way to help pupils understand and develop confidence in algebraic thinking and manipulation, as well as prepare them for its development in the secondary phase of education. In this way it will become a natural extension of earlier work in Primary rather than being a difficult topic that often remains a mystery throughout secondary education. Examples of our recommendations are given in our proposed curriculum in Section 4 above.

We would recommend the following be introduced <u>later</u> than stated in the draft National Curriculum:

There are two topics in particular that we feel strongly should be left until at least one year later.

Fractions
We see no reason to bring fractions into Year 1, where the emphasis should be on fully understanding the whole numbers from 0 to 20, giving a sound foundation for the concepts and extensions to come in later years. Year 2 seems a more appropriate time to begin formal work on fractions, since this is when pupils begin to build on their number sense, developed in Year 1.

Formal Written Methods

We welcome formal written methods to be covered in the National Curriculum but all the evidence from the MHPCs is that pupils first have to understand the concepts before moving to the formal vertical methods for basic operations of addition, subtraction, multiplication and division. We recommend that these methods are introduced at least one year later than given in the draft National Curriculum.

Additional Suggestions for Thematic Underpinning; as in the curricula for the MHPCs, we would also like to see the underpinning themes or concepts:

Classification, Enumeration, Problem solving, and *Logic and Rigour* should underpin the new syllabus. Illustrative examples are given below. The importance of using such supporting themes throughout the primary years cannot be over-emphasised. We need to help all our pupils become potential mathematical thinkers with the confidence to use their knowledge and skills in new situations and contexts.

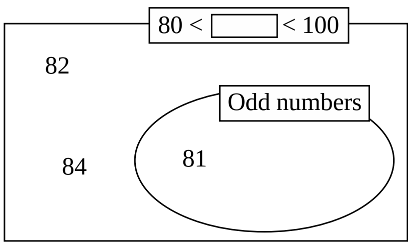
The underlying concepts that always feature in the seeing through of the curriculum in the MHPCs are not included as topics in our curriculum. We will look at each one separately here, even though they overlap. We give some illustrative examples below for specific years, with further examples given in Appendix 4 and answers to all the questions in Appendix 2.

Classification

Here we are dealing with methods of classifying numbers or shapes, etc; this type of analysis can be used throughout the primary phase both for understanding of topics and for reinforcement. There are a variety of ways of classifying but both two-way tables and Venn diagrams are particularly useful and straightforward.

Year 2

Which numbers make the inequality true?
Write them in the correct places on the diagram.



Enumeration

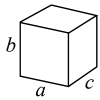
This fits well with classification but now we are counting, with real objects in early years and then moving to problems that require logical thought. The following example illustrates this approach.

Year 3

How many different **cuboids** can you build from 12 unit cubes?

a) Fill in the table.

b) Circle the cuboids which have at least one square face.



	Cuboids			
	1	2	3	4
Edge a =				
Edge b =				
Edge c =				

Problem Solving

This should underpin all aspects of mathematical development and can range from problems in context to those where pupils are expected to develop strategies, that is, to encourage pupils to think for themselves and take responsibility for their mathematical development. All the MHPCs use problem solving extensively as a means of getting their pupils to persevere and work independently or in groups (without teacher help) to find appropriate solutions. Problem solving should be part of the delivery of any curriculum; we include it here to ensure that its importance is not overlooked. Here is an illustrative example.

Year 5

Use each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 only once to make five whole numbers, so that one number is twice, another number is three times, another number is four times and the last number is five times the smallest number.

Logic and Rigour

This is the basis of all mathematical development but can so easily be overlooked in the focus on particular topics and methods. It is, though, of fundamental importance and is an essential part of mathematical development at all times. We give some examples of the type of problems which can be used to encourage the logical and rigorous thinking that we want to encourage in all our pupils.

Year 3

Make two 3-digit numbers using the numbers 0, 1, 3, 4, 5 and 8 so that:

a) their sum is the least possible, b) their sum is the greatest possible,

c) their difference is the least possible, d) their difference is the greatest possible.

Whilst these four themes develop naturally, their importance is in the underpinning of the curriculum, enhancing mathematical thinking throughout the stated syllabus.

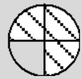
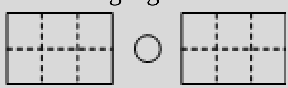
V
A First Curriculum for Mathematics
How to Tackle Maths Problems

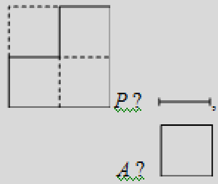
Our proposed primary curriculum for maths is based on the MHPC models adapted for the English context where the formal primary school starting age is 5+, whereas Japan and Singapore start at age 6+ and Finland at age 7+.

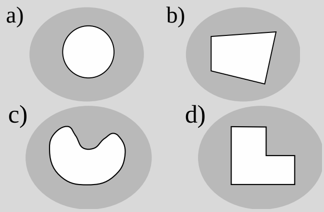
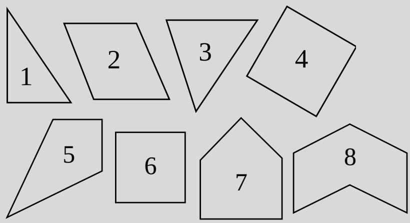
We consider Year 1 to be the essential time for pupils to master basic whole numbers from 0 to 20 in order to be in a position really to make progress in Year 2. We summarise below the main topics that we expect the majority of pupils, that is over 90 per cent of a normal intake, to master (in Japanese terms, ‘fix’) for each year in Primary school.

The Curriculum for Mathematics

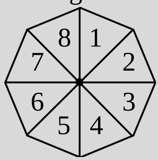
YEAR 1 (Age 5-6): ***By the end of the year, we expect pupils to be able to:***	***Examples***
Count to 20 and focus on whole numbers 0 to 20 on a number line, including additions and subtractions and using signs and symbols, including +, –, =. ≠, <, ≤, >, ≥ in simple calculations and know and use number bonds from 0 to 20. Recognise and name 2D shapes, including rectangles, squares, triangles and circles. Measure and compare lengths; tell the time to the hour and half past the hour; understand and use days, weeks, months and years.	7 + 4 = ? ? – 9 = 3 15 ≤ ? < 19 12 + ? = 12 11 + ? = 12 10 + ? = 1 … 0 ? = 12
YEAR 2 (Age 6-7): ***By the end of the year, we also expect pupils to be able to:***	***Examples***
Use whole numbers up to 100 and understand their place on a number line; focus on multiplication and division; understand place value; know, recall and use multiplication tables up to 10x10; use the correct order for operations.	36 + 18 = ? 72 – 15 = ? 0 x 8 = ? 1 x 8 =? 10 x 8 = ? 0 ÷ 8 = ? 8 ÷ 8 = ? … 80 ÷ 8 =? 28 ÷ 4 – 3 x 2 = ?

YEAR 2 (Age 6-7) ***continued***	
Introduce simple fractions, including halves and quarters.	There are 8 bananas in a bunch. 1 half bunch = ? bananas 1 quarter bunch = ? bananas What part of this circle is shaded? 
Extend the range of known shapes to include quadrilaterals, hexagons and pentagons; symmetry and mirror lines and similar shapes.	
Compare, estimate and measure for lengths (m, cm), mass (g, kg), capacity (l and cl) time (hours and minutes) and money (£ and p).	£1 – 10p = ? p 21 – ? cl = 20 cl 1 kg + 200 g = ? g
Classify the outcomes of experiments to be certain, possible but not certain, impossible.	When I throw a dice, which of the outcomes *Certain, Possible but not certain, Impossible* applies to each of the following: a) I will throw a 4 b) I will throw a 7 c) I will throw a number < 9
YEAR 3 (Age 7-8): ***By the end of the year, we also expect pupils to be able to:***	***Examples***
Extend numbers up to 1000; use negative numbers on an extended number line and in context (e.g. temperature, money); use column addition and subtraction; round numbers to the nearest 10 or 100.	Are the inequalities correct? Mark each one with a tick or a cross, and correct mistakes. -8 < -2, -6 > -7, 0 <-1, -10 > 2 List the whole numbers for which 60 is the nearest ten.
Recognise and use fractions in tenths; compare and simplify simple fractions.	2 fifths + ? fifths = 1 Colour the parts stated. Compare the two rectangles and write in the missing sign.  3 sixths 5 sixths
Solve simple equations and inequalities.	260 cm + 350 cm = 360 cm + ? cm Which positive whole number can be written instead of the shape: 936 + △ < 541 + 449

Recognise and name 3D shapes, including cuboids, cubes, cylinders, spheres and prisms; understand the concepts in 2D of horizontal, vertical, perpendicular, parallel and curved lines.	
Extend measures to length (mm, cm, m, km) capacity (ml, cl, l) and time (seconds, minutes, hours, days, months, years); calculate with money, in context; calculate the perimeter and area of simple 2D shapes.	Andrea had £6.42. She bought some flowers for £2.35. The money she has left is one third of the money her sister has. How much does her sister have?
Collect, interpret and present data in pictograms, bar charts and tables.	Count how many of the given units are in the perimeter and area of each shape. 
Calculate simple probabilities (with dice or coins)	Two boys and two girls had enough money for 1 ride in a 2-person dodgem car at the fair. They drew lots to see who would be the passenger and who would steer. What chance was there of the two girls riding together?
YEAR 4 (Age 8-9): *By the end of the year, we also expect pupils to be able to:*	***Examples***
Extend numbers up to 10,000; factors of numbers; prime numbers.	A cuboid is built from 72 unit cubes. How many units long can the edges be? First factorise 72 and then show the possibilities. Write down the whole numbers up to 100 that have only two factors.
Add and subtract simple fractions; equivalent fractions and simplest form. Understand decimals numbers with one or two decimal places; decimal form of simple fractions.	Polly bought 1.5 kg of apples and $\frac{5}{10}$ kg less of bananas. a)How many kg of bananas did she buy? b)How much fruit did she buy altogether?

YEAR 4 (Age 8-9) ***continued***	
Solve equations with fractions or decimal numbers; graph linear functions.	$5 + ? = 6.5, \quad ? + \frac{2}{7} = 1,$ $? - \frac{1}{5} = \frac{4}{5}$ Which numbers can be written instead of the letters: a) $400 \times 5 + a = 900$ b) $30 \times 20 + b > 6970$ c) $c \div 11 \geq 5$
Compare theoretical and experimental data.	Throw a dice 30 times but first predict the outcome. Compare your predictions with the actual outcome.
Understand the concepts of similarity, congruence, convex and concave for 2D shapes; plot points in 2D; compass directions.	In a dense forest there are some clearings. In which of the clearings could you hide from someone? Write a tick or a cross inside each one.  How many right angles do you turn through from a) N to W clockwise b) N to SE clockwise. List the polygons for which each statement is true. 

Convert between different measures for length, mass, capacity and time. Calculate perimeters and areas of composite shapes (rectangles and squares). Use tally charts to summarise data; interpret data given on charts and graphs.	a) It has a right angle. b) Every angle is a right angle. c) It has no right angles. d) It has an angle which is not a right angle. e) Every angle is a right angle but it is not a rectangle. 6.2 kg = ? g 520 cm = ? m ? cm = ? mm The perimeter of a triangle is 10 units. It has two equal sides. The length of each side is whole units. What is the length of each side?
YEAR 5 (Age 9-10): ***By the end of the year, we also expect pupils to be able to:***	***Examples***
Extend numbers up to 1,000,000; round numbers to the nearest 10, 100, 1000 etc; know prime and non-prime numbers (up to 100), prime factors, squares and square roots. Use formal written methods for multiplication and division. Convert fractions to/from decimals; calculate with expressions in fractional or decimals form. Solve linear equations or inequalities, including fractions and decimals and negative numbers. Measure angles, including right angles etc; recognize and compare different triangles, including isosceles, equilateral and right angled triangles; parallelogram, rhombus and trapezium. Understand the concepts of reflection, translation, rotation and enlargement of 2D shapes.	Write these fractions as decimals: a) $\frac{3}{2} = ?$ b) $\frac{6}{15} = ?$ c) $\frac{9}{8} = ?$

YEAR 5 (Age 9-10) ***continued***	
Calculate the volume and surface area of cuboids and composite shapes.	A box is shaped like a cuboid but is open at the top. Inside, it is 1.4 m long, 1 m wide and 80 cm high. What is its inner surface area? A = ? a) The area of the surface of a cube is 150 cm^2 .What is its volume in centimetre cubes (cm^3)? b) A cube is built from 64 1 cm cubes, so its volume is 64 cm^3. What is its surface area in centimetre squares (cm^3)?
Illustrate experimental data with bar charts. Calculate simple probabilities for experiments with dice, marbles and spinners etc; frequency and relative frequency.	The diagram shows a spinner used in a board game.  When the spinner is spun, what is the probability that it lands on: a) 1 b) 8 c) an even number d) a number less than 8 e) a number greater than 8 f) a number greater than 0?

YEAR 6 (Age 10+): ***By the end of the year, we also expect pupils to be able to:***	***Examples***
Multiply simple fractions; divide fractions by whole numbers; find fractions of quantities; understand and use simple percentages and convert between percentages, fractions and decimals; simple proportion problems.	A grocer had 1.8 kg of curry powder in stock. He sold 2 ninths of it on Monday and 30% of it on Tuesday. How much curry powder did the grocer have left? Express these fractions as decimals and percentages. Follow the example. a) $\frac{1}{5} = 0.2 \to 20\%$ b) $\frac{3}{5} = ?$ c) $\frac{1}{2} = ?$ d) $\frac{3}{2} = ?$
Evaluate simple formulae, including using numbers as fractions or decimals; number sequences.	Write a formula about the relationship of the given data. a) The **area** of a triangle with base *b* and height *h*. b) The **perimeter** of a regular octagon with side *a*: c) The **surface area** of a cuboid with edges *a*, *b* and *c*: d) The **volume** of a cuboid with edges *a*, 2*a* and 3*a*: e)The **surface area** of a cuboid with edges *a*, *a* and 2*a*.
Understand and use radius, diameter and circumference of a circle; 3D symmetry, including prisms; positions in full coordinate grid (all four quadrants).	If the statement is true, write 'T' if it is false, write 'F'. a) Every **isosceles triangle** has angles of 60°. b) No **rectangle** has adjacent equal sides. c) The **diameter** of a circle is twice the length of its radius. d) The **circumference** of a circle is its radius multiplied by π e) There is a **prism** which has congruent faces. f) A square-based **pyramid** has 5 vertices, 5 faces and 8 edges. g) If the diagonals of a quadrilateral **bisect** each other at right angles, the quadrilateral is a **rhombus**. h) A **tangent** to a circle can touch the circle at more than 1 point.

YEAR 6 (Age 10+) *continued*	
Know the relationship between common imperial units and metric measures. Be able to calculate the area and perimeter of parallelograms and triangles.	One side of a rectangle is 6.5 cm. Its area is 20.8 cm^2. What is the length of the adjacent side?
Use and interpret measures of central tendency, including mean, median and mode; range of set of data.	The heights of the 7 peaks in a mountain range are: 945 m, 1023 m, 1311 m, 996 m, 1286 m, 1504 m, 1150 m a) Write the data in increasing order. b) Calculate the difference between the highest and the lowest heights. The range of the sample is ? m. c) Calculate the average height of these 7 peaks. The mean of the sample is ? m. d) Find the middle value among the 7 heights. The range of the sample is ? m.
Estimate probabilities based on symmetry and on experimental data (relative frequency).	In a pack of 52 playing cards, there are **13** cards (Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King) in each of **4** suites: Diamonds and Hearts (*red*), Clubs and Spades (*black*). If you take a card from the pack at random, what is the probability that the card is: a) an Ace b) a 9 c) a Club d) a *red* card e) a Queen of Diamonds f) a Jack **or** a King of Spades g) an Ace of Clubs **or** a King of Hearts h) **not** an Ace?

For this curriculum, we would expect teachers to recall, revise and reinforce topics each year, but not to start again on the topics.

VI
Afterword

I have taught mathematics throughout my adult life, firstly to engineers at Sheffield University whilst a research student completing my PhD over 40 years ago; and since then to all levels from Reception to Masters Degree. The subject and its teaching still fascinate me and give me great pleasure, particularly when I find an improved way of explaining a topic or new contexts that use the mathematical topics under study.

This is particularly true of the mathematics in, for example, coding, algorithms, linear programming, critical path analysis – topics in what is now called Discrete Mathematics. I have gained so much enjoyment, inspiration, challenge and, at times, despair, from the subject, yet it is obvious that many of our school children (and their teachers!) do not experience even a fraction of this emotion and enjoyment but are bored with the subject.

This is regrettable and not helpful at a time when, in an increasingly technological world, we need young people who are confident, capable and able to use their mathematical skills and knowledge in new and challenging contexts.

I make no apology for concentrating on primary mathematics. Anything else at any other level is just 'putting sticking plaster on the cracks' of lack of understanding of the subject, rather than providing a long-term and sustainable solution. It is essential that pupils have a thorough understanding of the basics of mathematics before moving on to more advanced levels. This strong foundation will enable them to progress with confidence as they proceed through the curriculum.

We want our children to understand, appreciate and enjoy mathematics and to reach their potential in the subject: we want to see a nation at ease with mathematics. It is essential that a strong mathematics foundation is put in place at primary level to ensure that this can become a reality. This opportunity to develop and implement a revised National Curriculum for mathematics is a one-off chance to start to make a difference. Yes, there are many issues to be resolved in the implementation but first let us agree on a mathematical foundation that will provide the essential building blocks for future development and also be an enjoyable, relevant and challenging curriculum for both pupils and teachers!

I do hope that all concerned with the issues, that is, political parties, educationalists, teachers and parents, will come together to begin to develop a long-term solution to our mathematics problems. A new National Curriculum for primary mathematics is critical – but it is just the starting point!

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APPENDICES

APPENDIX 1: Japanese Lesson Study; Overarching aims and Specific Objectives

The following is an example of an **overarching aim**:

"Our students will become independent thinkers (learners) who enjoy working together to produce creative solutions in mathematics in unfamiliar situations."

and here are the corresponding specific objectives for the research lessons:

Enjoy doing mathematics *– to help students learn to enjoy and sense personal reward in the process of thinking, searching for patterns and solving problems*

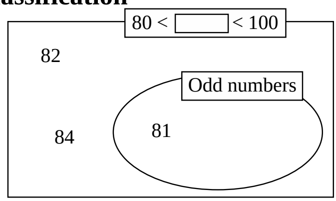
Gain confidence and belief in abilities *– to develop students' confidence in their ability to do mathematics and to confront unfamiliar tasks*

Be willing to take risks and to persevere *– to improve students' willingness to attempt unfamiliar problems and to develop perseverance in solving problems without being discouraged by initial setbacks.*

Interact with others to develop new ideas *– to encourage students to share ideas and results, compare and evaluate strategies, challenge results, determine the validity of answers and negotiate ideas on which they all can agree*

APPENDIX 2: Answers to illustrative questions in chapter 4

Classification



Even numbers:
86, 88, 90, 92,
94, 96, 98

Odd numbers:
83, 85, 87, 89, 91,
93, 95, 97, 99

Enumeration

	Cuboids			
	1	2	3	4
Edge *a* =	12	6	4	3
Edge *b* =	1	2	3	2
Edge *c* =	1	1	1	2

Cuboids 1 and 4 circled

Logic and Rigour

a) For example, 105 + 348 = 453
b) For example, 830 + 541 = 1371
c) 401 – 385 = 16
d) 854 – 103 = 751

Problem Solving

18, 36, 54, 72, 90

APPENDIX 3

Figure 1:

Algebraic Solution (See p.14)

The method given in the text makes such questions accessible to younger pupils. An algebraic method would be:

Present $A = \frac{1}{4}B$ or $B = 4A$ (a)

5 years' time $A + 5 = \frac{1}{3}(B + 5)$ or $B + 5 = 3(A + 5)$ (b)

Taking (a) from (b) gives

$$5 = 3(A + 5) - 4A$$

$$5 = 3A + 15 - 4A$$

$$5 = 15 - A$$

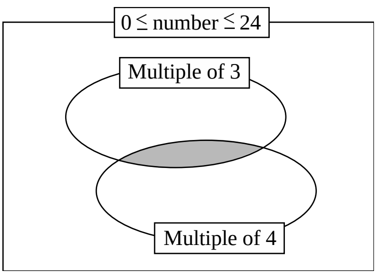
or $A = 15 - 5 = 10$

APPENDIX 4: Further examples of questions for specific years in the curriculum

Classification

Year 3

Write the whole numbers not less than 0 and not greater than 24 in the correct sets.

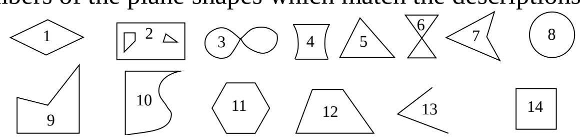


	Multiple of 3	Not a multiple of 3
Multiple of 4		
Not a multiple of 4		

What can you say about the numbers in the shaded areas?

Year 5

List the numbers of the plane shapes which match the descriptions.

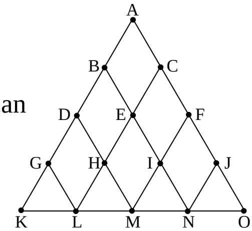


a) It is enclosed only by straight lines. b) It is enclosed by straight and curved lines.
c) It is enclosed only by curved lines. d) It is not enclosed.
e) It has parallel sides. f) It has perpendicular sides.
g) It has exactly 4 straight sides. h) It has exactly 6 vertices.

Ennumeration

Year 2

How many routes lead from A to K, L, M, N and O if you can only move down to the left or to the right?



Year 4

Write down 2-term additions using the numbers in *Set A*.

$$A = \{-3, 2, 1, 0, -5, 6\}$$

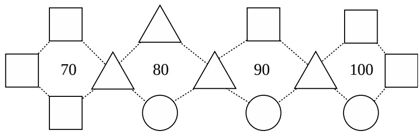
a) How many different additions are possible?

b) How many of the results are: i) positive ii) negative?

Problem Solving

Year 2

The same shape means the same number,
The sum of the four numbers at the corners equals the middle number.
Find the number for each shape.



Year 3

Which different 1-digit numbers could *a, b* and *c* be if $a + b + c = 14$ and $a \times b \times c = 84$.

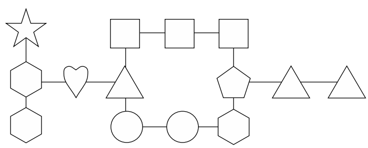
Year 6

A Year 6 class of 42 pupils took part in a special Physical Education lesson. The pupils could choose from basketball, swimming and gymnastics. We know that 20 of them did swimming, 19 did gymnastics and 18 played basketball. We also know that 7 pupils swam and played basketball, 8 pupils swam and did gymnastics and 6 pupils did gymnastics and played basketball. How many pupils took part in all three sports?

Logic and Rigour

Year 1

Divide the number 9 into three parts.
Do not use 0.
The same shape stands for the same number.



Year 5
Freddy Fox decided that from that day forward he would always tell the truth on Mondays, Wednesdays and Fridays but he would always tell lies on the other days of the week.
One day he said, "*Tomorrow I will tell the truth.*"
On which day of the week do you think he said this?

Year 6
We divided two numbers, 313 and 390, by the same 2-digit number. In each case, the remainder was the same. Which number could we have divided by?

Answers given in Appendix 5.

Appendices 5, 6 and 7 available to read online at www.politeia.co.uk/other/primary-mathematics-curriculum-appendices

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Today too many children finish primary school without the foundations of mathematics, and so cannot make progress in the subject at secondary school. As a result they are not equipped to lead and participate in a full adult life. *Primary Problems* emphasises the importance of mathematics in the primary years.

Its author, Professor David Burghes, considers the examples from a group of successful, mathematically high-performing countries, Singapore, Japan and Finland. He suggests that there are lessons for this country both in terms of what is taught and when. Professor Burghes compares the government's new proposals with these models, and while welcoming certain features, including the greater freedom for teachers, suggests where they might be modified for greater success.

He concludes his analyses with a clear outline curriculum, which will be welcomed as a model of clarity and brevity. It provides a year by year programme of study of what pupils should know and by what age.

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