David Burghes

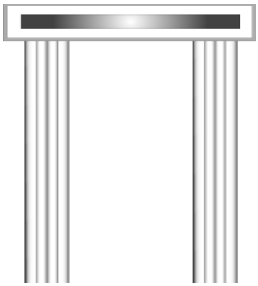
Primary Problems for the New Curriculum

Tougher Maths, Better Teachers

Curriculum Series
Edited by
Sheila Lawlor

POLITEIA

A FORUM FOR SOCIAL AND ECONOMIC THINKING



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Introduction
HMG's Proposal for Primary Maths: Can do better?

The draft National Curriculum for Primary Mathematics emphasises high levels of competence in basic skills. That emphasis should be welcomed in so far as it goes. In particular, it shares some of the high aspirations which characterise mathematically high performing countries (MHPCs), such as Finland, Japan and Singapore. Such a proposed level of competence is central to the success of primary mathematics.

Indeed, the examples of those countries suggest that an even greater emphasis than is proposed by the DfE should be put on the earlier mastery of some basic skills such as multiplication tables.

Greater emphasis should also be given to logical and mathematical thinking. In particular, algebraic concepts and elementary probability should be introduced and developed throughout the primary phase, rather than, as the DfE proposes, left until later. This would be in line with the mathematically high-performing countries which introduce algebra in the primary years with progression in concepts and continuity rather than a sudden jump on entering the Secondary phase. On account of algebra's importance to mathematical applications in the world today, there is probably more advantage to beginning earlier than later: this allows familiarity with the use of algebraic notation to develop with general mathematics progress throughout both phases. Indeed, algebraic knowledge also helps pupils enjoy mathematics, think mathematically and use mathematical skills in unfamiliar contexts.

To provide such a mathematical foundation, teachers will be needed with a high standard of mathematics. This points to higher entry standards in mathematics and dedicated continuing professional development with the focus on mathematical knowledge and concepts as well as teaching strategies. Both are common in mathematically high performing countries. Such a change would require some teachers to increase their own mathematical competence. Indeed the current emphasis on recruiting able maths graduates is a welcome development. However, many teachers would welcome the challenge to enhance their own mathematical knowledge through, for example, on-line interactive learning, together with face to face tutorials.

This response follows my earlier proposals for the primary curriculum, *Primary Problems: A First Curriculum for Mathematics*. It responds to the DfE's draft curriculum on a year by year basis, and suggests what ought to be included at an earlier stage, and what might be better left until later.* Such a course could, without complication, now be adopted in the official document.

To improve the nation's mathematical thinking over the longer term, strong foundations in mathematics at primary level are needed. Without this our pupils will be ill-prepared for secondary maths and schools will continue as now, merely to 'fire fight' in secondary and tertiary sectors.

* The response which follows comments on the DfE's proposed Programmes of Study on a year by year basis. Those topics which the DfE omits but which should be mastered are in blue; and those which should be introduced at a different stage are struck through in red. Readers may wish to compare with *Primary Problems: A First Curriculum for Mathematics*.

Year 1: Programme of Study

Number and place value ~~Pupils should be taught to:~~ Pupils should be ***able*** to:

- count to and across 100, forwards and backwards, beginning with 0 or 1, or from any given number
- count, read and write numbers to 100 in numerals, count in different multiples including ones, two, fives and tens
- given a number, identify one more and one less
- identify and represent numbers using concrete objects and pictorial representations including the number line, and use the language of: equal to, more than, less than (fewer), most, least
- read and write numbers from 1 to 20 in digits and words.

Addition and subtraction Pupils should be ***able*** to:

- read, write and interpret statements involving addition (+), subtraction (-) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- add and subtract one-digit and two-digit numbers to 20 (9 + 9, 18 – 9), including zero
- ***introduce and use the inequality signs, $<, \leq, >, \geq$ with numbers 0 to 20***
- solve simple one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems.

Multiplication and division Pupils should be ***able*** to:

- solve simple one-step problems involving multiplication and division, calculating the answer using concrete objects, pictorial representation and arrays with the support of the teacher.

Fractions : Leave until Year 2 ~~Pupils should be able to:~~

- ~~recognise, find and name a half as one of two equal parts of an object, shape or quantity~~
- ~~recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.~~

Measures Pupils should be ***able*** to:

- compare, describe and solve practical problems for:
  - lengths and heights (e.g. long/short, longer/shorter, tall/short, double/half)
  - mass or weight (e.g. heavy/light, heavier than/lighter than)
  - capacity/volume (full/empty, more than/less than, quarter)
  - time (quicker, slower, earlier, later)
- measure and begin to record the following:
  - lengths and heights
  - mass/weight
  - capacity and volume
  - time (hours, minutes, seconds)
- recognise and know the value of different denominations of coins and notes
- sequence events in chronological order using language such as: before and after, next, first, today, yesterday,
  tomorrow, morning, afternoon and evening
- recognise and use language relating to dates, including days of the week, weeks, months and years
- tell the time to the hour and half past the hour and draw the hands on a clock face to show these times.

Geometry: properties of shapes Pupils should be ***able*** to:

- recognise and name common 2-D and 3-D shapes, including:
  - 2-D shapes (e.g. rectangles (including squares), circles and triangles)
  - 3-D shapes (e.g. cuboids (including cubes), pyramids and spheres).

Geometry: position, direction, motion Pupils should be ***able*** to:

- order and arrange combinations of objects and shapes in patterns
- describe position, directions and movements, including half, quarter and three-quarter turns.

This is a helpful and well-thought-out Year 1 curriculum. However, we strongly suggest that fractions are left until Year 2 so that pupils can concentrate on the whole numbers from 0 to 20 and use, for example, the inequality signs alongside the equality signs. These form an important building block for algebraic concepts to come.

Note that we have changed "should be taught to" to "able to" to emphasis that this is a mastery curriculum and learners will often make progress through activities rather than being "taught". This change has been made in all years.

Year 2: Programme of Study

Number and place value ~~Pupils should be taught to:~~ Pupils should be ***able*** to:

- count in steps of 2, 3 and 5 from 0, and count in tens from any number, forward or backward
- recognise the place value of each digit in a two-digit number (tens, ones)
- identify, represent and estimate numbers using different representations, including the number line
- compare and order numbers from 0 up to 100; use <, > , $\leq$, $\geq$ and = , $\neq$ signs
- read and write numbers to at least 100 in numerals and in words
- use place value and number facts to solve problems.

Addition and subtraction Pupils should be ***able*** to:

- solve simple one-step problems with addition and subtraction;
  - using concrete objects and pictorial representation, including those involving numbers, quantities and measures
  - applying their increasing knowledge of mental and written methods
- recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100
- add and subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - a two-digit number and ones
  - a two-digit number and tens
  - two two-digit numbers
  - adding three one-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems.

Multiplication and division Pupils should be ***able*** to:

- ***recall and use multiplication and division facts for the 2, 3, 4, 5, 6, 7, 8, 9 and 10 multiplication tables, including recognising odd and even numbers***
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (x), division (÷) and equals (=) signs
- recognise and use the inverse relationship between multiplication and division in calculations
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve one-to-one problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Fractions Pupils should be ***able*** to:

- recognise, find, name and write fractions $\frac{1}{3}, \frac{1}{4}, \frac{2}{4}$ and $\frac{3}{4}$ of a length, shape, set of objects or quantity
- write simple fractions e.g. $\frac{1}{2}$ of $6 = 3$ and recognise the equivalence of two quarters and one half.

Algebra ***Pupils should be able to:***

- ***read and write number sequences.*** *See Appendix, Question 1*

Measures Pupils should be ***able*** to:

- choose and use appropriate standard units to estimate and measure length/height in any direction (m/cm); mass (kg/g); temperature; capacity (litres/ml) to the nearest appropriate unit, using rulers, scales, thermometers and measuring vessels
- compare and order lengths, mass, volume/capacity and record the results using >, < and =
- read relevant scales to the nearest numbered unit

- recognise and use symbols for pounds (£) and pence (p); combine amounts to make a particular value and match different combinations of coins to equal the same amounts of money; add and subtract money of the same unit, including giving change
- solve simple problems in a practical context involving addition and subtraction of money
- compare and sequence intervals of time
- tell and write the time to five minutes, including quarter past/to the hour and draw the hands on a clock face to show these times.

Geometry: properties of shapes Pupils should be ***able*** to:

- identify and describe the properties of 2-D shapes, including the number of sides and symmetry in a vertical line
- identify and describe the properties of 3-D shapes, including the number of edges, vertices, faces
- identify 2-D shapes on the surface of 3-D shapes, for example a circle on a cylinder and a triangle on a pyramid
- compare and sort common 2-D and 3-D shapes and everyday objects.

Geometry: position, direction, motion Pupils should be ***able*** to:

- order and arrange combinations of mathematical objects in patterns
- use mathematical vocabulary to describe position, direction and movements, including distinguishing between rotation as a turn and in terms of right angles for quarter, half and three-quarter turns (clockwise and anti-clockwise), and movement in a straight line.

Data Pupils should be ***able*** to:

- interpret and construct simple pictograms, tally charts, block diagrams and simple tables
- ask and answer simple questions by counting the number of objects in each category and sorting the categories by quantity
- ask and answer questions about totalling and compare categorical data.

Probability ***Pupils should be able to:***

- ***understand the use of the outcomes: certain, possible and impossible*** *See Appendix, Question 2*
- ***record outcomes of experiments with dice, marbles and coins.*** *See Appendix, Question 3*

Again this is a positive curriculum. During this year, pupils should be able to build on the foundation of whole numbers from Year 1 and cover all the multiplication tables up to 10 by 10. They are ready for this and for consistency and continuity, it is far better to 'fix' these facts early with continuing reinforcement and extension in later years. We also suggest early approaches to algebra as well as the introduction of simple probability.

Year 3: Programme of study

Number, place value and rounding ~~Pupils should be taught to:~~ Pupils should be ***able*** to:

- count from 0 in multiples of 4, 8, 50 and 100; finding 10 or 100 more or less than a given number
- recognise the place value of each digit in a three-digit number (hundreds, tens, ones)
- compare and order numbers up to 100
- identify, represent and estimate numbers using different representations
- read and write numbers to at least 1000 in numerals and in words
- solve number problems and practical problems involving these ideas.

Addition and subtraction Pupils should be ***able*** to:

- add and subtract numbers mentally, including
  - a three-digit number and ones
  - a three-digit number and tens
  - a three- digit number and hundreds
- add and subtract numbers with up to three digits, using the efficient written methods of columnar addition and subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Multiplication and division Pupils should be ***able*** to:

- ***recall and use multiplication and division facts up to and including 12 x 12 multiplication tables***
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to efficient written methods
- solve problems, including missing number problems, involving multiplication and division, including integer scaling problems and correspondence problems in which *n* objects are connected to *m* objects.

Fractions Pupils should be ***able*** to:

- count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10
- recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators
- recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators
- recognise and show, using diagrams, equivalent fractions with small denominators
- add and subtract fractions with the same denominator within one whole (e.g. $\frac{5}{7}+\frac{1}{7}=\frac{6}{7}$)
- compare and order unit fractions with the same denominator
- solve problems that involve all of the above.

Algebra ***Pupils should be able to:***

- ***solve simple equations and inequalities***. *See Appendix, Question 4*

Measures Pupils should be ***able*** to:

- measure, compare, add and subtract: lengths (m/cm/mm); mass (kg/g); volume/capacity (l/ml)
- measure the perimeter of simple 2-D shapes
- add and subtract amounts of money to give change, using both £ and p in practical contexts
- tell and write the time from an analogue clock, including using roman numerals from I to XII, and 12-hour and 24-hour clocks

- estimate the read time with increasing accuracy to the nearest minute; record and compare time in terms of seconds, minutes, hours and o'clock; use vocabulary such as a.m./p.m., morning, afternoon, noon and midnight
- know the number of seconds in a minute and the number of days in each month, year and leap year
- compare durations of events, for example to calculate the time taken by particular events or tasks.

Geometry: properties of shapes Pupils should be ***able*** to:

- draw 2-D shapes and make 3-D shapes using modelling materials; recognise 3-D shapes in different orientations; and describe them with increasing accuracy
- recognise angles as a property of shape and associate angles with turning
- identify right angles, recognise that two right angles make a half-turn, three make three quarters of a turn and four a complete turn; identify whether angles are greater than or less than a right angle
- identify horizontal, vertical, perpendicular and parallel lines in relation to other lines.

Data Pupils should be ***able*** to:

- interpret and present data using bar charts, pictograms and tables
- solve one-step and two-step questions such as 'How many more?' and 'How many fewer?' using information presented in scaled bar charts and pictograms and tables.

Probability ***Pupils should be able to:***

- ***calculate simple probabilities (with dice or coins).*** *See Appendix, Question 5*

We suggest some minor but important changes: having covered multiplication tables up to 10 by 10, we suggest reinforcing them here by continuing on to 12 by 12 as well as, crucially, continuing both the Algebra theme with simple equations and inequalities and the Probability theme with simple calculations, based on the symmetry idea and using fractions in a very meaningful context.

Year 4: Programme of study

Number, place value and rounding ~~Pupils should be taught to:~~ Pupils should be ***able*** to:

- count in multiples of 6, 7, 9, 25 and 100
- find 1000 more or less than a given number
- count backwards through zero in include negative numbers
- recognise the place value of each digit in a four-digit number (thousands, hundreds, tens, and ones)
- order and compare number beyond 1000
- identify, represent and estimate number using different representation
- round any number to the nearest 10, 100 or 1000
- solve number and practical problems that involve all of the above and with increasingly large positive numbers
- read Roman numerals to 100 (I to C) and understand how, over time, the numeral system changed to include the concept of zero and place value.

Addition and subtraction Pupils should be ***able*** to:

- add and subtract numbers with up to 4 digits using the efficient written methods of columnar addition and subtraction where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.

Multiplication and division Pupils should be ***able*** to:

- ~~reinforce and extend~~ multiplication and division facts for multiplication tables up to 12 x 12; use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal; written layout
- solve problems involving multiplying and adding, including using the distributive law and harder multiplication problems such as when *n* objects and connected to *m* objects.

Fractions Pupils should be ***able*** to:

- count up and down in hundredths; recognise that hundredths arise when dividing an object by a hundred and dividing tenths by ten
- solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number
- identify, name and write equivalent fractions of a given fraction, including tenths and hundredths
- add and subtract fractions with the same denominator.

Decimals and fractions Pupils should be ***able*** to:

- recognise and write decimal equivalents of any number of tenths or hundredths
- recognise and write decimal equivalents to $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$
- find the effect of dividing a one-digit or two-digit number by 10 and 100, identifying the value of the digits in the answer as units, tenths and hundredths
- round decimals with one decimal place to the nearest whole number
- compare numbers with the same number of decimal places up to two decimal places
- solve simple measure and money problems involving fractions and decimals to two decimal places.

Algebra ***Pupils should be able to:***

- ***solve equations with fractions or decimal numbers*** *See Appendix, Question 6*
- ***use and interpret simple formulae expressed in words*** *See Appendix, Question 7*
- ***understand how to graph simple linear functions.*** *See Appendix, Question 8*

Measures Pupils should be ***able*** to:

- convert between different units of measure (e.g. kilometre to metre; hour to minute)
- measure and calculate the perimeter of a rectilinear figure (including squares) in centimetres and metres
- find the area of rectilinear shapes by counting
- estimate, compare and calculate different measures, including money in pounds and pence
- read, write and convert time between analogue and digital 12- and 24-hour clocks
- solve problems involving converting from hours to minutes; minutes to seconds; years to months; weeks to days.

Geometry: properties of shapes Pupils should be ***able*** to:

- compare and classify geometric shapes, including quadrilaterals and triangles, based in their properties and sizes
- identify acute and obtuse angles and compare and order angles up to two right angles by size
- identify lines of symmetry in 2-D shapes presented in different orientations
- complete a simple symmetric figure with respect to a specific line of symmetry.

Geometry: position, direction, motion Pupils should be ***able*** to:

- describe positions on a 2-D grid as coordinates in the first quadrant
- describe movements between positions as translations of a given unit to the left/right, up/down
- plot specified points and draw sides to complete a given polygon.

Data Pupils should be ***able*** to:

- interpret and present discrete data using bar charts and continuous data using line graphs
- solve comparison, sum and difference problems using information presented in bar charts, pictograms, tables and simple line graphs.

Probability ***Pupils should be able to:***

- ***understand the difference between fair and unfair games*** *See Appendix, Question 9*
- ***understand the concept of predicted outcomes and compare with experimental outcomes in simple experiments.*** *See Appendix, Question 10*

This is a useful curriculum but, with multiplication tables already completed, we would want to see concepts in both Algebra and Probability developed further, using formulae to express relationships, and modelling the theoretical outcomes in simple experiments to compare with practical outcomes.

Year 5: Programme of study

Number, place value, approximation and estimation ~~Pupils should be taught to:~~ Pupils should be ***able*** to:

- read, write, order and compare number to at least 1 000 000 and determine the value of each digit
- count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000
- interpret negative numbers in context, count forwards and backwards with positive and negative whole numbers through zero
- round any number up to 1 000 000 to the nearest 10, 100, 11110, 10 000 and 100 000
- ***understand and use negative numbers in context and with reference to a number line***
- solve number problems and practical problems that involve all of the above
- read Roman numerals to 1000 (M) and recognise years written in Roman numerals.

Addition and subtraction Pupils should be ***able*** to:

- ~~add and subtract whole numbers with more than 4 digits including using efficient written methods (columnar addition and subtraction)~~
- ~~add and subtract numbers mentally with increasingly large numbers~~
- ***use negative numbers in simple calculations involving addition and subtraction***.
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Multiplication and division Pupils should be ***able*** to:

- identify multiples and factors, including finding all factor pairs
- solve problems involving multiplication and division where larger numbers are used by decomposing then into their factors
- know and use the vocabulary of prime numbers, prime factors and composite numbers
- establish whether a number up to 100 is prime and recall prime numbers up to 19
- multiply numbers up to 4 digits by a one- or two-digit number using an efficient written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally drawing upon know facts
- divide numbers up to 4 digits by a one-digit number using the efficient written method of short division and interpret remainders appropriately for the context
- multiply and divide whole numbers and those involving decimals by 10, 100 and 1000
- recognise and use square numbers and cube numbers, and the notation for squared and cubed
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

Fractions Pupils should be ***able*** to:

- compare and order fractions whose denominators are all multiples of the same number
- recognise mixed numbers and improper fractions and convert from one form to the other
- add and subtract fractions with the same denominator and related fractions; write mathematical statements >1 as a mixed number (e.g. $\frac{2}{5}+\frac{4}{5}=\frac{6}{5}=1\frac{1}{5}$)
- multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams.

Decimals and fractions Pupils should be ***able*** to:

- read and write decimal numbers as fraction (e.g. $(0.71 = \frac{71}{100})$)
- recognise and use thousandths and relate then to tenths, hundredths and decimal equivalents
- round decimals with two places to the nearest whole number and to one decimal place
- read, write, order and compare numbers with up to three decimal places
- solve problems involving number up to three decimal places.

Percentages, decimals and fractions Pupils should be ***able*** to :

- recognise the per cent symbol (%) and understand that per cent relates to "the number of parts per hundred", and write percentages as a fraction with denominator hundred, and as a decimal fraction
- solve problems which require knowing percentage and decimal equivalents of $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}, \frac{2}{5}, \frac{4}{5}$ and those with a denominator of a multiple of 10 or 25.

Algebra ***Pupils should be able to***

- ***solve linear equations with fractions, decimals, negative numbers*** *See Appendix, Question 11*
- ***solve linear inequalities.*** *See Appendix, Question 12*

Measures Pupils should be ***able*** to:

- convert between different units of measure (e.g. kilometre and metre; metre and centimetre; centimetre and millimetre; kilogram and gram; litre and millilitre)
- understand and use basic equivalences between metric and common imperial units and express them in approximate terms
- measure and calculate the perimeter of composite rectilinear shapes in centimetres and metres
- calculate and compare the are of squares and rectangles including using standard units, square centimetres and square metres and estimate the area of irregular shapes
- recognise and estimate volume (e.g. using $1\ \text{cm}^3$ blocks to build cubes and cuboids) and capacity (e.g. using water)
- solve problems involving converting between units of time
- solve problems involving addition and subtraction of units of measure (e.g. length, mass, volume, money) using decimal notation.

Geometry: properties of shapes Pupils should be ***able*** to:

- identify 3-D shapes, including cubes and cuboids, from 2-D representations
- know angles are measured in degrees; estimate and measure them and draw a given angle, writing its size in degrees
- identify:
  - multiples of $90°$
  - angles at a point on a straight line and $\frac{1}{2}$ a turn (total $180°$)
  - angles at a point and one whole turn (total)
  - reflex angles, and
  - compare different angles
- draw shapes using given dimensions and angles
- state and use the properties of a rectangle (including squares) to deduce related facts
- distinguish between regular and irregular polygons based on reasoning about equal sides and angles

Geometry: position, direction, motion Pupils should be ***able*** to:

- identify, describe and represent the position of a shape following a reflection or translation, using the appropriate language, and know that the shape has not changed.

Data Pupils should be ***able*** to:

- solve comparison, sum and difference problems using information presented in line graphs
- complete, read and interpret information in tables, including timetables.

Probability ***Pupils should be able to:***

- ***calculate theoretical probabilities for outcomes with dice, number wheels, spinners and identify probabilities on a 0 -1 number line*** *See Appendix, Question 13*
- ***find relative frequency in simple experiments.*** *See Appendix, Question 14*

We recommend the continued development of the Algebraic and Probability themes alongside Number, Geometry and Measures and we also suggest the introduction, in context, of negative numbers in both simple applications and in calculations.

Rather than focusing on addition and subtracting of larger and larger numbers for mathematical development, it is important by this year to concentrate on the concepts that underpin mathematics, not just numeracy. Hence our suggested inclusion of negative numbers as well as the further development of Algebra and Probability in this year.

Year 6: Programme of study

Number, place value and rounding ~~Pupils should be taught to:~~ Pupils should be ***able*** to:

- read, write, order and compare numbers up to 10 000 000 and determine the value of each digit
- round any whole number to a required degree of accuracy
- use negative numbers in context, and calculate intervals across zero
- solve number problems and practical problems that involve all of the above.

Addition, subtraction, multiplication and division Pupils should be ***able*** to:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the efficient written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the efficient written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- ***multiplication and division of negative numbers***
- perform mental calculation, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry our calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculation and determine, in the context of a problem, levels of accuracy.

Fractions Pupils should be ***able*** to:

- use common factors to simplify fractions; use common multiples to express fractions in the same denomination
- compare and order fractions, including fraction >1
- associate a fraction with division to calculate decimal fraction equivalents (e.g. 0.375) for a simple fractions (e.g. $\frac{3}{8}$)
- add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions
- multiply simple pairs of proper fractions, writing the answer in its simplest form (e.g. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$)
- divide proper fractions by whole numbers (e.g. $\frac{1}{2} \div 2 = \frac{1}{4}$).

Decimals and fractions Pupils should be ***able*** to:

- identify the value of each digit to three decimal places and multiply and divide numbers by 10, 100 and 1000 where the answers are up to three decimal places
- multiply one-digit numbers with up to two decimal places by whole numbers
- use written division methods in cases where the answer has up to two decimal places
- solve problems which require answers to be rounded to specified degrees of accuracy.

Percentages, decimals and fractions Pupils should be ***able*** to:

- solve problems involving the calculation of percentages of whole numbers or measures such as 15% of 360 and the use of percentages for comparison
- recall and use equivalences between simple fractions, decimals and percentage, including in different contexts.

Ratio and proportion Pupils should be ***able*** to:

- solve problems involving the relative sizes of two quantities, including similarity
- solve problems involving unequal sharing and grouping.

Algebra Pupils should be ***able*** to:

- express missing number problems algebraically
- use simple formulae expressed in words
- generate and describe linear number sequences
- find pairs of numbers that satisfy number sentences involving two unknowns.
- ***use, graph and interpret simple formulae expressed in words*** *See Appendix, Question 15*
- ***solve linear inequalities involving decimals and fractions and negative numbers.*** *See Appendix, Question 16*

Measures Pupils should be ***able*** to:

- solve problems involving the calculation and conversion of units of measure, using decimal notation to three decimal places where appropriate
- use, read, write and convert between standard units, converting measurements of length, mass, volume and time from a smaller unit of measure to a larger unit, and vice versa, using decimal notation to three decimal places
- convert between miles and kilometres
- recognise that shapes with the same areas can have different perimeters and vice versa
- calculate the area of parallelograms and triangles
- recognise when it is necessary to use the formulae for area and volume of shapes
- calculate, estimate and compare volume of cubes and cuboids using standard units, including centimetre cubed (cm^3) and cubic metres (m^3) and extending to other units, such as mm^3 and km^3.

Geometry: properties of shapes Pupils should be ***able*** to:

- recognise, describe and build simple 3-D shapes, including making nets
- compare and classify geometric shapes based on their properties and sizes and find unknown angles in any triangles, quadrilaterals, and regular polygons
- illustrate and name parts of circles, including radius, diameter and circumference
- find unknown angles where they meet at a point, are on a straight line, and are vertically opposite.

Geometry: position, direction, motion Pupils should be ***able*** to:

- describe positions on the full coordinate grid (all four quadrants)
- draw and translate simple shapes on the coordinate plane, and reflect them in the axes.

Data Pupils should be ***able*** to:

- interpret and construct pie charts and line graphs and use these to solve problems
- calculate and interpret the mean as an average.

Probability *Pupils should be able to:*

- ***use the language associated with probability such as certain, equally likely, unlikely, impossible and use this to describe the likelihood of particular events*** *See Appendix, Question 17*
- ***calculate theoretical probabilities based on symmetry*** *See Appendix, Question 18*
- ***understand the concept of relative frequency and use it to estimate probabilities .*** *See Appendix, Question 19*

As in Year 5, our suggestions would help develop the logical mathematical thinking of all pupils so that by the end of Key Stage 2, as with pupils in mathematically high-performing countries, they would have strong mathematical foundations to build on in the Secondary sector as well as the ability to perform fast, efficient and accurate numerical calculations.

III
Conclusion
Tougher Maths, Better Teachers

If the new primary mathematics curriculum is to be implemented successfully, mathematics teachers will need to be equipped with the necessary mathematical subject knowledge and pedagogical skills.

A major initiative will be needed to improve mathematics teaching and learning in the primary sector. To begin with, current teachers could be encouraged to take A and AS level mathematics examinations. Teachers could also work with local teaching schools, enhancing pedagogy through the use of Japanese style lesson study and with input from the increasing number of expert teachers being trained in Primary Mathematics. Greater use of new technology should also be considered.

Over the longer term we must recognize that change is needed; so too are expert and enthusiastic teachers. Without a significant enhancement of mathematics teaching in UK primary schools, difficulties in and obstacles to secondary and tertiary mathematics will continue with implications for the economic well being of the country. Long term solutions must be found. They must start in the Primary sector with a strong mathematical foundation for our pupils as happens in the mathematically high performing countries.

IV
Appendix

Examples of concepts to be introduced

1. Continue the sequences. Complete the rules.

a) 0, 4, 8, ...

2, 6, 10, ...

3, 7, 11, ...

The sequences **increase** by ....

2. I am going to toss a coin once. How certain can I be of the result?
Join up the statements on the left to the correct labels on the right.

I will throw a head.	Certain
I will throw a tail.	Possible, but not certain
I will throw a head **and** a tail.	
I will throw a head **or a tail.**	Impossible

3. Throw a dice on your desk 10 times. Keep a tally of the numbers thrown in the table. Fill in the last column to show how often you threw each number.

	Throws										
	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	Total
1											
2											
3											
4											
5											
6											

4. a) Which number does each letter represent? Fill in the missing numbers.

$5 \times a = 25$ $\quad a =$ $\qquad$ $7 \times b = 42$ $\quad b =$ $\qquad$ $c \times 4 = 36$ $\quad c =$

b) List the numbers which make the inequality true.

$70 \div 5 >$ ☐ $> 200 \div 10$ ☐ : .....

5. I have 3 bags of marbles. Bag A contains 10 marbles, Bag B contains 20 marbles and Bag C contains 30 marbles. One marble in each bag is *red.*

Which bag gives me the best chance of picking the *red* marble?

6. Solve the equations.

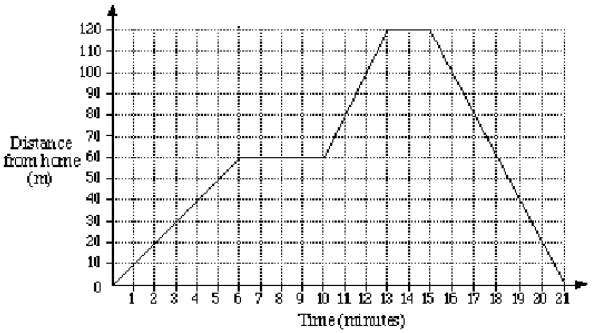
a) $5 +$ ☐ $= 6.5$ $\qquad$ b) ☐ $+ 3.9 = 5.7$ $\qquad$ c) ☐ $+ \frac{1}{4} = \frac{5}{4}$

7. We want to rearrange some books on two bookshelves.

At the moment, there are 156 books on the bottom shelf and on the top shelf there are 30 books more than there are on the bottom shelf.

Rearrange the books so that there are the same number of books on both shelves.

8. *Tammy Tortoise* went for a walk from her house to the field and back again. The graph shows how far she was from home during that time.



a) How far away from home did *Tammy* go?

b) For how long was she away from home?

c) When did she start her return journey?

d) How many times did *Tammy* stop to rest?

9. Tom has a bag with 7 BLUE balls and 1 WHITE ball in it.

Steve has a bag with 6 BLUE balls and 2 WHITE balls in it.

They each close their eyes and take a ball from their own bag until one of them takes out a WHITE ball. This person is the winner.

Is this game fair?

10. Predict the results for each outcome first, then do the experiment.

Throw a dice 20 times and keep a tally of how it lands in this table.

	Prediction	**Tally of 20 throws**	**Totals**
⚀			
⚁			
⚂			
⚃			
⚄			
⚅			

How many times did you get:

a) a 2 **or** a 3 b) less than 5 c) not less than 5 .

11. Which integers can be written instead of the question marks?

a) $-12+2\times ?=-16$ $?=$

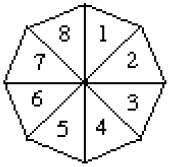
b) $?\div(+3)=-6$ $?=$

12. List the integers represented by the question marks. Show the solutions on a number line.

a) $?\times 6\geq -18$ $?=$

b) $+8\times ?\leq 0$ $?=$

14. The diagram shows a spinner used in a board game.



When the spinner is spun, what is the probability that it lands on:

a) 1 b) 8

c) an even number d) a number less than 8

e) a number greater than 8 f) a number greater than 0?

15. The volume of a **square-based pyramid** can be calculated using this formula:

$$V = \frac{A \times h}{3}$$

where A is the area of the base and h is the height of the pyramid.

How high is the pyramid if its base edge is 36 cm and its volume is $17289\ \text{cm}^3$?

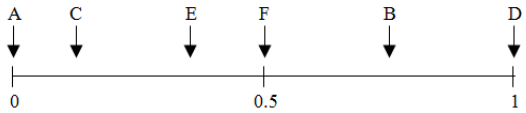
16. Solve the inequalities. Check that your solutions are correct.

a) $a-(-4) > -2$

b) $\frac{4}{5}+(-b) \le \frac{3}{10}$

c) $c+(+4) \ge +4$

17. The probability scale shows the probability of 6 outcomes: A, B, C, D, E and F.



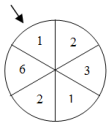
a) Which outcomes is: i) **certain** to happen ☐ ii) **impossible** ☐

iii) the **most unlikely** to happen but is not impossible? ☐

b) Which outcomes are **more likely than C** to happen? ☐

c) Which outcome is the **least likely** to happen, but is not impossible? ☐

18. The spinner is fairly divided into 6 equal sectors but the possible outcomes do not have equal chances.



a) List the possible outcomes.

b) Calculate the probability of each outcome.

19. Throw a fair dice **60** times. Keep a tally of the outcomes. Write the **frequency** in the table and calculate the **relative frequency** of each outcome.

Outcome	Tally of 60 throws	Frequency	Relative Frequency
⚀			
⚁			
⚂			
⚃			
⚄			
⚅			
		$n = 60$	

What do you estimate will be the probability of throwing ⚅?

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Good mathematics begins in primary school. Ministers should be congratulated for demanding high levels of competence in basic skills in the new primary maths curriculum, says Professor David Burghes. But, as *Primary Problems for the New Curriculum* suggests, the curriculum could be tougher.

Professor Burghes draws on the experience of the high performing countries, Finland, Japan and Singapore, to suggest improvements in the final curriculum. Basic skills, such as multiplication tables, should be mastered earlier than is proposed. There should be greater emphasis placed on logical and mathematical thinking. Some algebra and probability should be introduced in the primary years. Ultimate success will, however, depend on the recruitment to primary teaching of a higher proportion of mathematically able and competent teachers.

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